

① First Mean Value Theorem for Integrals

$$f \in \mathcal{R}[a, b]$$

$$\Rightarrow \exists c \in (a, b) \mid \int_a^b f(x) dx = f(c) \cdot (b-a).$$

② Second Mean Value Theorem for Integrals

Given $f \in \mathcal{R}[a, b]$, w monotone on $[a, b]$.

$$\exists c \in [a, b] \mid$$

$$\int_a^b w(x) f(x) dx = w(a) \int_a^c f(x) dx + w(b) \int_c^b f(x) dx.$$

③ Integration by Parts

Let f, g be differentiable on $[a, b]$.

$$\text{b.t. } f', g, g' \in \mathcal{R}[a, b].$$

$\therefore$ By Fundamental Theorem of Calculus.

$$\int_a^b (fg)'(x) dx = (fg)(b) - (fg)(a)$$

$$=: (fg)(x) \Big|_a^b$$

$$\Rightarrow \int_a^b [(fg)'(x) + (f'g)(x)] dx = (fg)(x) \Big|_a^b$$

$$2) \int_a^b (fg')(x) = \left[(fg)(x) \right]_a^b - \int_a^b (gf')(x)$$

Which is often stated as:

$$\int_a^b uv = \left[u \int v \right]_a^b - \int_a^b [u' \int v]$$

Some details have been omitted.